

Mixing of axino and goldstino, and axino mass

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Axino, related to the SUSY transformation of axion, can mix with goldstino in principle. This case is realized when some superfields carrying nonvanishing Peccei-Quinn charges develop both scalar VEVs and F-terms. In this case, we present a proper definition of axion and axino. With this definition, we present the QCD axino mass in the most general framework, including non-minimal Kähler potential. The axino mass is known to have a hierarchical mass structure depending on accidental symmetries. With only one axino, if $G_A = 0$ where $G = K + \ln|W|^2$, we obtain $m_{\tilde{a}} = m_{3/2}$. For $G_A \neq 0$, the axino mass depends on the details of the Kähler potential. In the gauge mediation scenario, the gaugino mass is the dominant axino mass parameter. Therefore, we can take the theoretical QCD axino mass as a free parameter in the study of its cosmological effects, ranging from eV to multi-TeV scales, without a present knowledge on its ultraviolet completion.

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I. INTRODUCTION

The lightest supersymmetric particle (LSP) with R-parity conservation is absolutely stable and can contribute to the present energy density of the universe as a dominant component of cold dark matter (CDM). If the QCD axion solves the strong CP problem, its fermionic SUSY partner, *axino*, must be considered in the CDM estimate in cosmology. Firstly, it can be a natural candidate for CDM if it is the LSP [1, 2]. Second, if the axino is much heavier than the neutralino LSP, still it affects the mass density estimate of the neutralino LSP because the nonthermal decay of axino may dominate the estimate [3]. Therefore, it is of utmost importance to clarify what is the axino mass in a specific supergravity model. The axino mass is obtained when SUSY is broken. A naive guess on the axino mass is of order $m_{3/2}$ since the axino mass is the soft term. But a leading loop corrections can reduce it to $\alpha m_{3/2}$. If this is suppressed, the next level hierarchical masses are arising from the gauge mediation and even some accidental symmetries can reduce it further to $m_S(f_a/M_P)$, $m_S(f_a^2/M_P^2)$, \dots [5]. In fact, in the literature, axino mass has been considered in a vast mass regions from eV to trans-TeV [1–15].

The axino mass depends on two symmetry breakings, the Peccei-Quinn (PQ) symmetry breaking and SUSY breaking. The effect of the PQ symmetry breaking must introduce a massless axion except from the contribution of the anomaly term and the effect of SUSY breaking introduces a mass parameter $m_{3/2}$. The contribution to the axino mass from the parameter $m_{3/2}$ is arising by the F-terms while the PQ symmetry breaking is given by the VEVs of scalar fields. In this paper, we study these two contributions in a most general form, and express the axino mass in terms of $m_{3/2}$ with the general Kähler form.

In global SUSY models, the axion a is defined through the Peccei-Quinn(PQ) transformation for $N_{\text{DW}} = 1$ fields

[17],

$$\begin{aligned} f_a e^{ia/f_a} &= \sum_i v_i e^{ia_i/f_i} \\ a &\propto \sum_i v_i \Gamma_i \frac{a_i}{f_a}, \end{aligned} \quad (1)$$

where Γ_i, v_i and a_i are the eigenvalue of the PQ charge operator Γ , the vacuum expectation value(VEV) and the phase of ϕ_i , respectively. The PQ direction of a_i is $\Gamma_i \theta = \Gamma_i \frac{a}{f_a}$.

A prototype axion model with a global SUSY needs at least three chiral fields, to introduce a VEV breaking the PQ symmetry [16]. In this introduction, we do not introduce SUSY breaking, but only introduce the PQ symmetry and its breaking. The superpotential having a global PQ symmetry is

$$W = R(S_1 S_2 - f_a^2), \quad (2)$$

where the PQ charges of R, S_1 , and S_2 are 0, +1, and -1, respectively. Equation (2) has an additional R -symmetry whose charges are 2, 0, and 0, respectively for R, S_1 , and S_2 . To have the standard kinetic energy terms, the Kähler potential is taken as $K = RR^* + S_1 S_1^* + S_2 S_2^*$. Then, the potential V is given by

$$V = |S_1 S_2 - f_a^2|^2 + |R S_2|^2 + |R S_1|^2. \quad (3)$$

which is minimized at $\langle S_1 S_2 \rangle = f_a^2$ and $\langle R \rangle = 0$. To show the superTrace (STr) of M^2 , we choose the fields near $\langle S_1 \rangle = \langle S_2 \rangle = f_a$,

$$\begin{aligned} S_1 &= \frac{1}{\sqrt{2}}(\sqrt{2}f_a + \rho_1 + ia_1) \\ S_2 &= \frac{1}{\sqrt{2}}(\sqrt{2}f_a + \rho_2 + ia_2) \\ R &= \frac{1}{\sqrt{2}}(\rho_R + ia_R), \end{aligned} \quad (4)$$

where ρ 's are scalars and a 's are pseudoscalars. The mass matrix for CP even scalars (ρ_R, ρ_1, ρ_2) and CP odd scalars (a_R, a_1, a_2) are given by the same squared mass matrix,

$$f_a^2 \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad (5)$$

so eigenvalues are given by $(2f_a^2, 2f_a^2, 0)$. One CP odd scalar should be massless since it is the goldstone boson corresponding to the spontaneously broken PQ symmetry.

On the other hand, fermion mass matrix is given by

$$f_a \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (6)$$

whose eigenvalues are $(\sqrt{2}f_a, -\sqrt{2}f_a, 0)$. Supersymmetry is not broken since $\langle V \rangle = 0$, so should produce $\text{STr } M^2 = 0$,

$$\text{STr } M^2 = m_{\text{CP even}}^2 + m_{\text{CP odd}}^2 - 2m_{\text{fermion}}^2 = 0. \quad (7)$$

In Eq. (6), the smaller mass is the axino mass, and one of the larger ones is the hypothetical goldstino mass. This larger one becomes exactly massless when the superHiggs mechanism is operative as will be discussed later.

The superpotential Eq. (2) can be rewritten in terms of R , the superfields $S = (S_1 + S_2)/\sqrt{2}$, and the axion superfield $A = (S_1 - S_2)/\sqrt{2}$ containing axion $(a_1 - a_2)/\sqrt{2}$,

$$W = R \left(\frac{S^2 - A^2}{2} - f_a^2 \right) \rightarrow -\frac{1}{2}RA^2 \rightarrow 0. \quad (8)$$

The VEVs are $\langle R \rangle = \langle A \rangle = 0$ and $\langle S \rangle = \sqrt{2}f_a$. After integrating out R , there does not exist low energy self interactions of A due to the nonrenormalization theorem.

Here note that the zero mass eigenstates of scalar, pseudoscalar and fermion indicate that they are related by SUSY transformation and form the axion supermultiplet A . In general, the same mass eigenstates of scalar, pseudoscalar and fermion are not related by supersymmetry transformation and do not form a supermultiplet. For the supermultiplet condition, interactions must be supersymmetric also. In our case, however, the zero mass eigenstates form a supermultiplet A which survives down to the low energy scale. This renders the nonlinear representations for $S_1 = \varphi e^{A/f_a}$ and $S_2 = \varphi e^{-A/f_a}$, which show explicitly the shift symmetry of the axion superfield.

II. THE PQ SYMMETRY IN SUPERGRAVITY

The supersymmetrization of axion models introduces a full axion supermultiplet A which contains the pseudoscalar axion a , its scalar partner *saxion* s , and their

fermionic partner axino \tilde{a} ,

$$A = \frac{1}{\sqrt{2}}(s + ia) + \sqrt{2}\tilde{a}\vartheta + F_A\vartheta\vartheta, \quad (9)$$

where F_A stands for an auxiliary field and ϑ for a Grassmann coordinate.

In the supersymmetric version of axionic models, the interactions of the saxion and the axino with matter are related by supersymmetry to those of the axion. Saxion and axino are better to accompany a in Eq. (1) to preserve SUSY. In Eq. (1), f_a is a VEV of some real scalar field which is called φ_R . Supersymmetrization of φ_R needs its pseudoscalar partner φ_I , forming a complex scalar φ . When the PQ symmetry is not broken, the PQ charged fields are of the φ type, its phase changes when the PQ transformation is performed, and the fields are the real and the imaginary components of φ . On the other hand, if it is spontaneously broken, the PQ charge is not realized unitarily but realized in the Nambu-Goldstone manner, *i.e.* the Goldstone boson a is created, goes up to the phase as in Eq. (1), and the PQ symmetry is its shift symmetry. [The PQ symmetry is nonlinearly realized on the action to a .] A nonzero SUSY breaking F -term of the φ type fields signals the PQ symmetry breaking also if it carry PQ charges. Therefore, the direction of axion does not necessarily coincide with the direction of axino which is going to be orthogonal to the goldstino which in turn is determined by the F -terms. In this paper, we define the axion and axino properly, and set up the formulae for the axino mass even in case that a mixing of axino with goldstino is present. Here, the axion component is still defined by the coefficient of ϑ^0 term since the F -term or the coefficient of ϑ^2 term is auxiliary.

A. Origin of axino-goldstino mixing

The PQ symmetry (as any global symmetries) in supergravity has a meaning if both the superpotential W and the Kähler potential K respect the symmetry. Expansion of the Kähler potential in powers of $1/M_P$ leads to the following type,

$$K = \sum_{I,J} f_I(\{\phi_i\})g_J(\{\phi_j^*\}) + \text{h.c.} \quad (10)$$

where $\{\phi_i\}$ and $\{\phi_j^*\}$ are sets of holomorphic and anti-holomorphic fields, respectively. From the Kähler potential, one can obtain its contribution to V as

$$V \in \int d^2\vartheta \int d^2\bar{\vartheta} \sum_{I,J} f_I(\{\Phi_i\})g_J(\{\bar{\Phi}_j\}) + \text{h.c.} \quad (11)$$

where $\{\Phi_i\}$ and $\{\bar{\Phi}_j\}$ are the superfields corresponding to $\{\phi_i\}$ and $\{\phi_j^*\}$, respectively.

Consider the leading term beyond the minimal term in K , for example $(1/M_P)H_u H_d X^*$. This must preserve the

PQ symmetry so that X carries the PQ charge $\Gamma(X)$ as the sum of $\Gamma(H_u)$ and $\Gamma(H_d)$. Namely, the PQ symmetry is also broken by an F term, *i.e.* X_F^* of X^* [19]. This can be obtained from the superpotential as

$$W \sim X_1 X_2 X \quad (12)$$

where $\Gamma(X_1) + \Gamma(X_2) = -(\Gamma(H_u) + \Gamma(H_d))$. In this case, $\mu = -X_1 X_2 / M_P$ is obtained in Ref. [20]. This example shows that it is sufficient to consider the PQ charge carrying scalar components to pick up the axion component, and should not include the PQ charge carrying F-terms for a definition of the axion. Otherwise, we double count some components.

1. Introduction of φ type fields

In non-supersymmetric case, as axion is defined in Eq. (1), and its property determined by the $U(1)_{\text{PQ}}$ symmetry. In the Wigner-Weyl (WW) realization of the PQ symmetry, the PQ charged fields transform as

$$\begin{aligned} \Phi_i &\rightarrow e^{i\Gamma_i \theta} \Phi_i \\ \Gamma \Phi_i &= \Gamma_i \Phi_i \end{aligned} \quad (13)$$

where $\Gamma_i = -i\partial/\partial\theta$. In the Nambu-Goldstone (NG) phase, there appears a Goldstone boson a which can be a combination of the original phase fields. Let a be in Φ such that in the WW phase it is expanded as

$$\Phi = \sum_i c_i X_i. \quad (14)$$

In the NG phase, the probability amplitude for a to be in the phase of X_i is $c_i = v_i/V_a$ where $V_a = \langle \Phi \rangle$ and $v_i = \langle X_i \rangle$. The PQ operation on Eq. (14) is

$$\Gamma_a \Phi = \sum_i c_i \Gamma_i X_i. \quad (15)$$

When the PQ symmetry is realized in the NG manner by giving X_i its VEV v_i , then the charge operator Γ is not unitarily realized, and then we must use the shift symmetry of the phase fields a , instead of the PQ symmetry operation Γ , with the original information on the eigenvalues Γ_i . So, we apply the infinitesimal shift symmetry on $\sum_i c_i X_i$, $\delta a = f_a \delta\theta$ and $\delta a_i = \Gamma_i f_a \delta\theta$, and we use the relation proportional to $\delta\theta$ to code the spontaneous symmetry breaking. For $\Phi \sim e^{i\Gamma_a a/f_a}$ where f_a is determined by the axion-gluon-gluon anomaly term, then by acting the shift operation on Φ of Eq. (15) in the NG phase we obtain $\Gamma_a \Phi$ as $\sum_i (v_i/V_a) \Gamma_i X_i$ where c_i is v_i/V_a which is the probability amplitude for a to be in X_i . In $\delta a_i = \Gamma_i f_a \delta\theta$, Γ_i encodes the original degeneracy number of the phase of X_i as the axion field a completes one period in the NG phase. Representing $\Phi = (V_a + \rho_\perp) e^{i\Gamma_a a/f_a}$ and $X_i = (v_i + \rho_{i\perp}) e^{i\Gamma_i a/f_a}$, where ρ_\perp is in the perpendicular direction to axion's Mexican

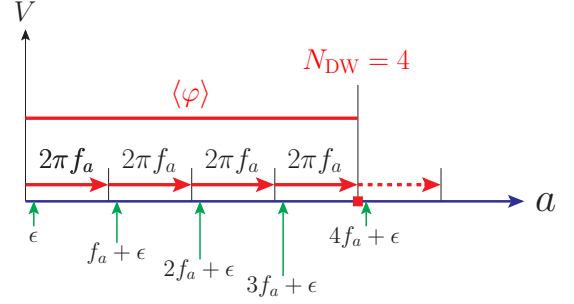


FIG. 1: The definition of N_{DW} in the Nambu-Goldstone phase. Here, $N_{\text{DW}} = 4$.

hat valley and has mass proportional to V_a . Since the $\rho_{i\perp}$ masses are different, they should belong to different superfields. From this discussion, we obtain the axion superfield A in

$$\Gamma_a \varphi_A e^{A/f_a} \equiv \sum_i \frac{v_i}{V_a} \Gamma_i \varphi_i e^{A/f_a} \quad (16)$$

where $c_i = v_i/V_a$ is used. Note that we used e^{A/f_a} on both sides in view of Fig. 1 since the axion shift of $2\pi f_a$ is fully accounted for by e^{A/f_a} . Any integer cannot be multiplied or divided in the exponent. Note that φ_A is composed of two real fields ρ_\perp and $\text{Im} \varphi_A$, so is φ_i , and their VEVs are $\langle \varphi_A \rangle = V_a$ and $\langle \varphi_i \rangle = v_i$. Also, A is composed of two real fields s and a , and its VEV is vanishing $\langle A \rangle = 0$.

In the NG phase, we must state the domain wall number by the axion shift. In Fig. 1, $N_{\text{DW}} = 4$ is schematically shown. The decay constant f_a is given by the coefficient of axion-gluon-gluon anomaly [17]. So, the axion potential returns to itself by a shift of $2\pi f_a$ [18]. The original domain wall number is given in Fig. 1 by the length $\langle \varphi \rangle$.

Functions of the Nambu-Goldstone fields are multivalued. In the original field space, the field returns to itself after the a shift of $2\pi N_{\text{DW}} f_a$. If two axion directions have two domain wall numbers n_1 and n_2 , the multiplicity of the vacua is the least common divisor of n_1 and n_2 [18]. Thus, we can write $n_1 = N_{\text{DW}} \beta_1$ and $n_2 = N_{\text{DW}} \beta_2$, where β_1 and β_2 are relatively prime. So, Eq. (16) can be written as

$$\frac{1}{N_{\text{DW}}} \varphi_A e^{A/f_a} \equiv \sum_i \frac{v_i}{V_a} \frac{1}{N_{\text{DW}} \beta_i} \varphi_i e^{A/f_a} \quad (17)$$

Therefore, we obtain

$$a = \frac{\sum_i a_i / \beta_i}{\sqrt{\sum_i \beta_i^{-2}}} \quad (18)$$

whose literary form has the domain wall number 1 since $\{\beta_i\}$ do not have a common divisor. From (17), we obtain

$$V_a^2 = \sum_i \frac{v_i^2}{\beta_i}. \quad (19)$$

Now, consider the special case Eq. (2) where in terms of three fields an ultraviolet completion is achieved, $W = R(S_1 S_2 - f_a^2)$ [16]. With the complete knowledge on W of Eq. (2), A and φ are obtained in terms of S_1 and S_2 as $S_1 = \varphi e^{A/f_a}$ and $S_2 = \varphi e^{-A/f_a}$ [22]. Writing the low energy field A in terms of the high energy fields R, S_1 and S_2 is not of much use in the region where only the low energy effective fields and the PQ quantum numbers of the original fields are known. In most cases, the information on φ type fields is not needed.

2. Appearance of A type fields in W

For the Kähler potential, we can have any function of $A + A^*$. But, a supersymmetric W with A is

$$W(A) = 0. \quad (20)$$

The $U(1)_{\text{PQ}}$ invariance guarantees that the axion superfield A does not appear in the superpotential. But, below the SUSY breaking scale we can introduce the soft terms in W by introducing the auxiliary field Θ and respecting the shift symmetry,

$$\begin{aligned} W &= M^3 \Theta e^{\alpha A/f_a} \\ \Theta &= 1 + m_{\text{SUSY}} \vartheta^2. \end{aligned} \quad (21)$$

where M is a parameter and $m_{\text{SUSY}} = m_S$ is the parameter describing the SUSY breaking soft terms. Then, $\ln|W|^2$ appearing in local asusy is invariant under the shift of a , with A defined in Eq. (16).

If there are more spontaneously broken global $U(1)_{A_i}$ ($i = 1, 2, \dots$) symmetries, coaxions a_i with the decay constant f_i must respect the shift symmetries and we must consider the following W ,

$$W = M^3 \Theta e^{\alpha A/f_a} \prod_{i=1,2,\dots} e^{\alpha_i A_i/f_i}. \quad (22)$$

3. Comments on $N_{\text{PQ}} \text{MSSM}$

In the literature, models based on $N_{\text{PQ}} \text{MSSM}$ have been considered [23, 24].

In Ref. [23], the original fields were used to show the existence of light field X_{ew} which must correspond to our φ type field in the effective low energy theory framework. Also, by considering the original fields in a specific model, it was shown that φ, φ^2 and φ^3 are not present. In this effective theory, the PQ symmetry is broken, and the φ type fields do not carry the PQ quantum number. So, $(\mu + \varphi)H_u H_d$ are the allowed interaction. At the fundamental level, the μ term arises from the original fundamental fields, dictated by the PQ symmetry [20]. Counting the number of degrees of freedom, we double the fields S_1 and S_2 to $\varphi_1 e^{A_1}$ and $\varphi_2 e^{A_2}$. As commented before, the light A type field is $A = A_1 - A_2$. $A_1 + A_2$ becomes heavy. For the φ type fields, only X_{ew} is light and

the one orthogonal to it becomes heavy. So, at low energy, we have the exponential field A and the φ type field X_{ew} . X_{ew} does not accompany a phase field. The coefficient of exponential of A is very large, and A does not appear explicitly in the superpotential. Axino and saxion are in A . So after integrating out our double counted fields, we end up with a superfield X_{ew} and a superfield A . So W of Eq. (2) has the same degrees at low energy in the NG phase. This is the way to write down the low energy theory corresponding to Eq. (2). How X_{ew} couples to the other light fields depends on the ultraviolet completion. Note also that the axion interaction depends on axion models [25, 26].

On the other hand, Ref. [24] considered $\varphi, \varphi^2, \varphi^3$ and $\varphi H_u H_d$ terms without the μ term.

4. Comments on the model-independent axion

The model-independent axion in superstring models is combined with the dilaton to make a supermultiplet [27],

$$D = \frac{1}{g^2} + i \frac{a_{MI}}{8\pi M_P} \rightarrow s + \frac{f_{MI}}{8\pi} e^{ia_{MI}/f_{MI}} \quad (23)$$

where $f_{MI} \sim 10^{16}$ GeV [28], and $\langle s \rangle \simeq 2M_P$ is not the φ type field. Because the corresponding $U(1)$ is gauged, a_{MI} is absorbed to the $U(1)$ gauge boson, and the $U(1)$ symmetry remains as a global PQ symmetry below the scale f_{MI} . Only for this anomalous model-independent axion in string models, there is no accompanying φ type field. Below f_{MI} , the resulting pseudo-Goldstone boson will accompany a φ type field. We speculate that this model independent axion is the only place for the axion not accompanying its φ type field.

B. Goldstino, axion and axino

The axion component is defined in Eq. (1). So, whatever the non-vanishing PQ charge carrying F-terms are, the axion is properly defined only by the PQ charge carrying ϑ^0 terms. However, the nonvanishing F-terms define the goldstino component.

Supersymmetry is spontaneously broken when the potential has nonzero VEV, $\langle V \rangle = \sum_i F^i F_i > 0$ where $F^i \equiv K^{i\bar{j}} F_{\bar{j}}$. Then, there should be a massless fermion, goldstino. In supergravity, it is absorbed to the longitudinal component of gravitino ψ_μ through the super-Higgs mechanism. The goldstino superfield, to which goldstino belongs, can be defined by

$$Z = \sum_i \frac{F^i}{F} X_i, \quad (24)$$

where $F = \sqrt{\sum_i F^i F_i}$ which becomes the F-term of Z . Among X_i , the axion superfield is defined by the PQ charges of X_i . All the other chiral fields orthogonal to A

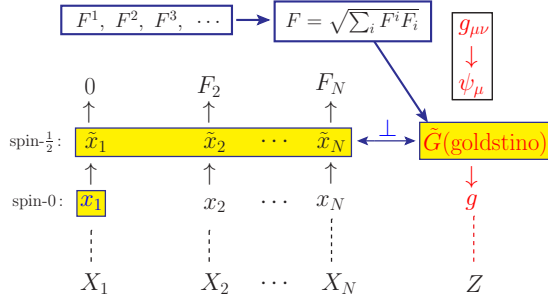


FIG. 2: The case for more than one axino. It is ambiguous to choose the partner of the QCD axion. Normalization toward the canonical kinetic term is not depicted in the figure.

are called *coaxino* directions. Then, we can consider two cases in which the axion superfield A allows

- $F_A \neq 0$, or
- $F_A = 0$, but $F^A \neq 0$ from Kähler mixing with other SUSY breaking fields.

This case is shown in Fig. 2.

With the canonical Kähler potential, F^i is just F_i^* , and the goldstino fermion gives the SUSY breaking direction exactly. On the other hand, in the presence of a Kähler mixing, F^i is nonzero even though $F_i = 0$ and seems to contribute to the goldstino. However, the physical goldstino, defined by the zero mass eigenstate with the canonical kinetic term, does not contain such a state. This can be shown as follows. In supergravity, fermion kinetic and mass terms are given by

$$e^{-1}\mathcal{L} = -iG_{i\bar{j}}\bar{\psi}^{\prime\bar{j}}\bar{\sigma}_\mu\mathcal{D}^\mu\psi^{\prime i} + \frac{1}{2}m'_{ij}\psi^{\prime i}\psi^{\prime j} + \frac{1}{2}m^{\dagger}_{i\bar{j}}\bar{\psi}^{\prime\bar{i}}\bar{\psi}^{\prime\bar{j}} \quad (25)$$

where \mathcal{D}_μ is a general covariant derivatives and $m'_{ij} = m_{3/2}[\nabla_i G_{\bar{j}} + (1/3M_P^2)G_i G_{\bar{j}}]$ is the fermion mass with the goldstino field moded out. The primed fields and the primed mass matrix are in the interaction basis. To obtain the physical states, we first make the kinetic terms canonical and then diagonalize the mass matrix. With the redefinition of $\psi^{\prime i} = V^i_a \psi^a$, Eq. (25) is written as

$$\begin{aligned} e^{-1}\mathcal{L} &= -i[V^\dagger_{\bar{a}}{}^{\bar{j}}G_{i\bar{j}}V^i_b]\bar{\psi}^{\bar{a}}\bar{\sigma}_\mu\mathcal{D}^\mu\psi^b \\ &+ \frac{1}{2}[V^T m' V]_{ab}\psi^a\psi^b + \frac{1}{2}[V^\dagger m'^\dagger V^*]_{\bar{a}\bar{b}}\bar{\psi}^{\bar{a}}\bar{\psi}^{\bar{b}} + \dots \\ &= -i\bar{\psi}^{\bar{a}}\bar{\sigma}_\mu\mathcal{D}^\mu\psi^a + \frac{1}{2}m_{ab}\psi^a\psi^b + \frac{1}{2}m^{\dagger}_{\bar{a}\bar{b}}\bar{\psi}^{\bar{a}}\bar{\psi}^{\bar{b}} + \dots \end{aligned} \quad (26)$$

Requiring the canonical kinetic terms as $V^\dagger_{\bar{a}}{}^{\bar{j}}G_{i\bar{j}}V^i_b = \delta_{\bar{a}b}$, or $V^T_b{}^i G_{i\bar{j}} V^{*\bar{j}}_{\bar{a}} = \delta_{b\bar{a}}$, the mass in this basis is $m_{ab} = [V^T m' V]_{ab}$ which we have to diagonalize.

The matrix V can be written in the form of $V = US$ where U is a unitary matrix and S is a scaling matrix.

Taking the inverse of the canonical kinetic term condition, $V^T_b{}^i G_{i\bar{j}} V^{*\bar{j}}_{\bar{a}} = \delta_{b\bar{a}}$, we obtain

$$\begin{aligned} (V^{*-1})^{\bar{a}}_{\bar{j}} G^{\bar{j}i} (V^{T-1})_i{}^b \\ = (S^{-1}U^T)^{\bar{a}}_{\bar{j}} G^{\bar{j}i} (U^* S^{-1})_i{}^b = \delta^{\bar{a}b}. \end{aligned} \quad (27)$$

Taking the complex conjugation, we obtain

$$(S^{-1}U^\dagger)^a{}_j G^{j\bar{i}} (US^{-1})_{\bar{i}}{}^{\bar{b}} = \delta^{a\bar{b}}, \quad (28)$$

or equivalently,

$$(S^{-1}U^\dagger)^a{}_j G^{j\bar{i}} = (SU^\dagger)^{a\bar{i}} \quad (29)$$

On the other hand, m'_{ij} has zero eigenvalue in the direction of $G^i = G^{i\bar{j}} G_{\bar{j}}$. Since $m' = (V^T)^{-1} m V^{-1} = (U^* S^{-1}) m (S^{-1} U^\dagger)$, $m'_{ij} G^j = 0$ implies $m_{ab} (S^{-1} U^\dagger)^b{}_j G^j = 0$. So, $(S^{-1} U^\dagger)^b{}_j G^{j\bar{k}} G_{\bar{k}}$ is the goldstino direction. Plugging Eq. (29) into this, we obtain

$$(S^{-1}U^\dagger)^a{}_j G^{j\bar{k}} G_{\bar{k}} = (SU^\dagger)^{a\bar{j}} G_{\bar{j}}. \quad (30)$$

We have rotated fermion as $\psi^{\prime i} = V^i_a \psi^a$ to make the kinetic term canonical. So, in the new (physical) basis, $\psi^a = (V^{-1})^a{}_i (\psi')^i = (S^{-1} U^\dagger)^a{}_i (\psi')^i$. Note that S is a real scaling diagonal matrix. Rotation of the direction is determined by U . SUSY breaking direction G_i defined in the interaction basis ψ' is rotated by U in the new basis ψ . Then $U^\dagger{}^{a\bar{i}} G_{\bar{i}}$ is just supersymmetry breaking direction of $G_{\bar{i}}$, not G^i .

As indicated within the yellow box in Fig. 2, there is an ambiguity in identifying the mass eigenstate axino corresponding to the QCD axion.

When there exists only one axino, there is no ambiguity in identifying the mass eigenstate QCD axino, because it must be the orthogonal state to the goldstino. For two fermions, we can consider two cases separately as shown in Fig. 3 for $F_A = 0$ and Fig. 4 for $F_A \neq 0$. Even for one axino and goldstino case, the one beyond the axion multiplet is called the coaxino as indicated in the left-hand side of Figs. 3 and 4 with a thick brown bar. These relatively simple cases will be discussed explicitly in Sec. III A where we calculate the axino mass by the gravity mediation. In the regrouping of Figs. 3 and 4, we violated the supermultiplet condition, $QA = \tilde{a}$, and may make an error in estimating the axino mass by an order $O(m_{3/2}^2/M_P)$.

The goldstino multiplet Z is defined by the total F^* term,

$$Z = \frac{F^A}{F} A + \sum_{i \neq A} \frac{F^i}{F} X_i \equiv \frac{F^A}{F} A + \frac{F^C}{F} C, \quad (31)$$

where C is the sum of SUSY breaking coaxino, orthogonal to the axino superfield. Their bosonic and fermionic components are not the components of the original superfields. Since axion and goldstino are finally defined

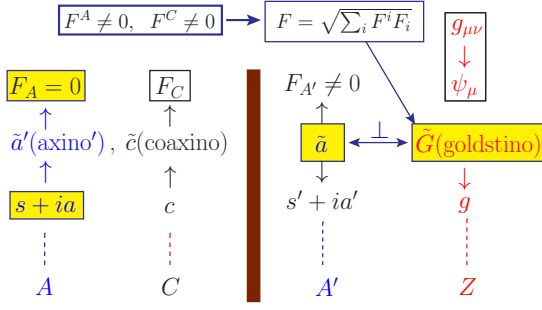


FIG. 3: The case $F_A = 0$ for the axion (blue) and goldstino (red) multiplets. The axion direction a is defined by the PQ symmetry through Eq. (9) and the goldstino (\tilde{G}) and axino (\tilde{a}) directions are defined by the fermion mass eigenvalues. Normalization toward the canonical kinetic term is not depicted in the figure.

after SUSY is broken, the mismatch is generic. In view of Figs. 3 and 4 introducing one coaxino, we can regroup the axion and coaxino multiplets as

$$\begin{aligned} A &= (A, \tilde{a}', F_A) \\ C &= (c', \tilde{c}, F_C) \end{aligned} \quad (32)$$

where the scalar component of the axion multiplet is $A = s' + ia$. a' in Figs. 3 (and also in 4) is not a mass eigenstate since the identification of $\tilde{a} \perp \tilde{G}$ does not care about its superpartner a' . The scalar potential is given by

$$V = M_P^2 e^{G/M_P^2} [G^{i\bar{j}} G_i G_{\bar{j}} - 3M_P^2], \quad (33)$$

and pseudoscalar a in A does not appear in V since G is a function of $(1/2)(A + A^*) = s'$. It is a massless eigenstate, and therefore the pseudoscalar mass matrix is already diagonalized in the left-hand side of the brown bar. On the other hand, the fermion mass matrix should be diagonalized in the right-hand side of the brown bar. One eigenvalue in the direction of F^i , (F^A, F^C) is massless along the direction of Z which is goldstino. After making the fermion kinetic term canonical, goldstino indicates the direction (F_A, F_C) . The remaining eigenvalue is interpreted as the axino mass. If $F_A \ll F_C$, $\tilde{a} \simeq \tilde{a}' - (F^A/F)\tilde{Z}$ and the mass eigenstate is axino-like.

In general, however, after the SUSY breaking, the mass eigenstates are

$$\begin{aligned} \text{Scalar : } & s, \text{ Re } c \\ \text{Pseudoscalar : } & a, \text{ Im } c = \text{Im } c' \\ \text{Fermion : } & \tilde{a} = \tilde{a}' - (F^A/F)\tilde{Z}, \tilde{Z}, \end{aligned} \quad (34)$$

where s and c are the mass eigenstates after diagonalizing the mass matrix in the (s', c') basis. In the last step, the shift symmetry of a must be invoked to guarantee the massless axion.

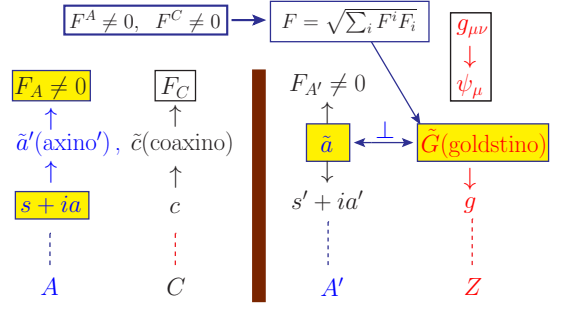


FIG. 4: The same as Fig. 3 except for $F_A \neq 0$.

This process makes sense when we consider the axion interaction with gluino

$$\frac{1}{f_a} \int d^2\vartheta A \mathcal{W}^\alpha \mathcal{W}_\alpha. \quad (35)$$

If $F_A = 0$ as in Fig. 3, the gluino mass does not get a contribution from the axion multiplet. However, if $F_A \neq 0$, the gaugino mass should be studied carefully in a specific model. This is true also if the Kähler potential has a large mixing term.

In our language, after the goldstino \tilde{G} is absorbed to ψ_μ , we can write the ψ_μ interaction as[21],

$$\frac{m_{3/2}}{2} \psi_\mu \sigma^{\mu\nu} \psi_\nu = \int d^2\vartheta \frac{G}{2\sqrt{3}M_P} \psi_\mu \sigma^{\mu\nu} \psi_\nu. \quad (36)$$

So, before hiding the goldstino into ψ_μ , any chiral field interaction in supergravity has a coupling suppressed by $1/M_P$,

$$\frac{(\text{coupling}) \Phi'_i}{M_P}. \quad (37)$$

Therefore, the coupling of A' is suppressed by M_P , not by f_a , in Eq. (35). After absorbing the goldstino into ψ_μ , any other chiral fields are orthogonalized not to have an F-term but its supergravity coupling is suppressed by $1/M_P$ again, and the goldstino superfield G couples to a gauge supermultiplet as

$$\frac{(\text{coupling})}{M_P} \int d^2\vartheta Z \mathcal{W}^\alpha \mathcal{W}_\alpha \quad (38)$$

where $F_G = F$ gives the gaugino mass. In addition, the axino interaction with gluino is

$$\begin{aligned} & \frac{1}{f_a} \int d^2\vartheta A \mathcal{W}^\alpha \mathcal{W}_\alpha \\ & \rightarrow \frac{1}{f_a} (\tilde{a} - \epsilon_a \tilde{G}) (\text{gluino})^a (\text{gluon})^a \\ & \rightarrow \frac{1}{f_a} \tilde{a} (\text{gluino})^a (\text{gluon})^a \\ & \quad - \epsilon_a \frac{1}{f_a M_P} (\partial^\mu \psi_\mu) (\text{gluino})^a (\text{gluon})^a. \end{aligned} \quad (39)$$

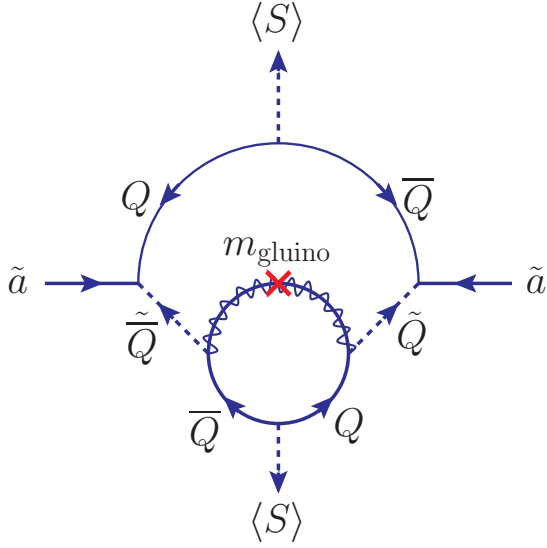


FIG. 5: The two-loop axino mass in the KSVZ model.

So axino interaction implies some accompanying gravitino interaction, suppressed by the product $f_a M_P$. This makes sense since it must respect the spontaneous symmetry breaking suppressed by f_a and the super-Higgs mechanism suppressed by M_P .

C. Axino mass with SUSY diagrams

In Eq. (6), the smaller mass was shown as the axino mass, and one of the larger ones is the hypothetical goldstino mass. This larger one becomes exactly massless when the superHiggs mechanism is operative as discussed in the previous section. This phenomenon can be compared to the positive complex scalar mass splitting into one Higgs boson mass plus a goldstone boson mass when the Higgs mechanism is in operation. The correspondence is what the axino mass to the goldstino is what the Higgs boson mass to the goldstone boson mass.

But there is also a non-gravitational contribution to the axino mass. Even if the axion is massless, both fermion partners are massive as shown in Eq. (6). This is a supersymmetric mass where fermionic masses are split differently from those of bosons. As an example, consider a KSVZ coupling

$$W = -f_Q S \bar{Q} Q. \quad (40)$$

There exists a two-loop diagram generating the axino mass as shown in Fig. 5. The mass estimate is

$$m_{\tilde{a}} \sim \frac{f_Q^2 g_c^2}{(4\pi^2)^2} m_{\text{gluino}} \simeq \frac{\alpha_{f_Q} \alpha_c}{\pi^2} m_{\text{gluino}}. \quad (41)$$

So, the axino mass contains the contribution

$$m_{\tilde{a}} = \sum_{a=\text{gaugino}} \xi_a m_{1/2,a} + \dots \quad (42)$$

where $m_{1/2,a}$ are the gaugino masses. These contributions give masses of order up to probably 10 GeV.

In the gravity mediation scenario, there exists another parameter $m_{3/2}$ which can be much larger than 10 GeV. Without the axino-goldstino mixing in the Kähler potential, the axino direction is the same as that of axion and the superpotential determines the axino mass. It means that axino mass arises from loop diagrams as in the Fig. 5 example. Therefore, without the axino-goldstino mixing in the Kähler potential, axino mass is not going to be larger than 10 GeV. Thus, a very heavy axino mass is possible only if there is a significant $A - Z$ mixing in the Kähler potential.

If we consider the anomaly mediation, there would be additional contributions from breaking the conformal symmetry which appear as anomalous dimensions in the superpotential terms [29]. Generally, they are proportional to $\gamma_{\text{anom } I} m_{3/2}$ for the term W_I in the superpotential. Thus, the axino mass has contributions from all the above cases and expressed as

$$m_{\tilde{a}} = \left(\xi_{\text{goldstino}} + \sum_{I=\text{terms in } W} \xi_I^{\text{anom}} \right) m_{3/2} + \sum_{a=\text{gaugino}} \xi_a m_{1/2,a}. \quad (43)$$

We expect that ξ_I^{anom} is of order $O(10^{-2})$. An example has been discussed in [30].

D. With accidental symmetries

In [5], the possibility of keV axino was discussed in case the superpotential has an accidental symmetry. The keV [9] and even eV [8] range axino masses are possible with some accidental symmetries. The accidental symmetries may forbid the leading order masses of the scales $m_{3/2}$ and $m_{1/2,a}$.

In the gauge mediation scenario, $m_{3/2}$ is negligible and the axino-goldstino mixing does not give a significant contribution. Then, the loops may give the dominant contribution. But the accidental symmetry may forbid diagrams of the form of Fig. 5. The superpotential may introduce a nonrenormalizable term suppressed by M_P , and the expansion parameter is $f_a/M_P \sim 10^{-7}$. Thus, the axino mass of Fig. 5 is further suppressed by $\sim 10^{-7}$ and we expect $10 \text{ GeV} \cdot 10^{-7} \simeq 1 \text{ keV}$. If it is further suppressed, then the estimated axino mass is of order 10^{-3} eV .

In the gravity mediation scenario, $m_{3/2}$ is a TeV scale and the axino mass depends on the Kähler potential. Without the axino-goldstino mixing, it was commented in Subsec. II C, and the discussion with some accidental symmetries is the same as the above paragraph.

III. PARAMETRIZATION WITH NONMINIMAL KÄHLER FORM

Chun and Lukas studied the axino mass with the minimal Kähler form [10]. Here we go beyond the minimal Kähler form, work with the PQ symmetry realized in the NG manner, and include the effects of F terms of the PQ charged fields which affect the axino component.

Equation (16) is our definition of axion superfield, and the Kähler potential must respect the shift symmetry of axion. Therefore, the lowest order terms in the Kähler potential with some mixing with SUSY breaking coaxino C are¹

$$K = \frac{1}{2}(A + A^*)^2 + \epsilon(A + A^*)(C + C^*) + CC^* + M(A + A^*). \quad (44)$$

The SUSY breaking can be introduced in terms of Kim's generalized form [16] of the Polonyi one [31]. Here, however, we will parametrize the SUSY breaking just by an auxilliary holomorphic constant Θ ,

$$\Theta = 1 + m_S \vartheta^2. \quad (45)$$

In view of the discussion of Subsubsec. II A 2, if there are coaxions then the superpotential can be taken as

$$W(C) = \frac{C^4}{M_P} \Theta + \dots \quad (46)$$

with $\langle W(C) \rangle = M^3 \sim (10^{13} \text{ GeV})^3$.

A. Local SUSY with one axino

The most important requirement is that goldstino is defined in the vanishing cosmological constant(CC) vacuum, satisfying the U(1) invariance condition. Thus, in calculating the axino mass, we satisfy the following three conditions:

(i) The vanishing CC condition,

$$G^{i\bar{j}} G_i G_{\bar{j}} = 3M_P^2, \quad (47)$$

where where $G = K + M_P^2 \ln |W|^2$.

(ii) The vacuum stabilization condition,

$$G^{j\bar{k}} G_{\bar{k}} \nabla_i G_j + G_i = 0. \quad (48)$$

(iii) For the U(1) invariance condition, we use

$$\begin{aligned} K &= K(A + A^*, C, C^*) \\ W &= \Theta e^{\alpha A/f_a} W(C). \end{aligned} \quad (49)$$

If there are more than one coaxino, we have

$$W = W(C) e^{\alpha A/f_a} \times e^{\alpha A_1/f_1} \times \dots \quad (50)$$

The superpotential in (49) preserves the shift symmetry of A since in $G = K + \ln |W|^2$, the $|W|^2$ part is read as $|W|^2 = |W(C)|^2 \Theta e^{\alpha(A+A^*)/f_a}$.

Now, let us calculate the axino mass given by

$$m = m_{3/2} [\nabla_i G_j + \frac{1}{3} G_i G_j] \quad (51)$$

for two classes of $\langle C \rangle = 0$ and $\langle C \rangle \neq 0$.

1. Case for $G_A = 0$ and $G^A \neq 0$

As an example for $G_A = 0$ but for $G^A \neq 0$, we consider

$$\begin{aligned} K &= \frac{1}{2}(A + A^*)^2 + CC^* + \epsilon(A + A^*)(C + C^*) \\ W &= e^{\alpha A/f_a} W(C) \end{aligned} \quad (52)$$

which is an example for $G_A = 0$ but $G^A \neq 0$; so goldstino has some axino component. From $G = K + M_P^2 \ln |W/M_P^3|^2$, we obtain

$$\begin{aligned} G_A &= (A + A^*) + \epsilon(C + C^*) + \frac{\alpha}{f_a} \\ G_C &= C^* + \epsilon(A + A^*) + M_P^2 \frac{W_C}{W} \end{aligned} \quad (53)$$

from which the Kähler metric elements and their inverse elements are given by

$$\begin{aligned} G_{AA} &= 1, \quad G_{AC} = \epsilon, \quad G_{CA} = \epsilon, \quad G_{CC} = 1, \\ G^{A\bar{A}} &= \frac{1}{1 - \epsilon^2}, \quad G^{A\bar{C}} = -\frac{\epsilon}{1 - \epsilon^2}, \\ G^{C\bar{A}} &= -\frac{\epsilon}{1 - \epsilon^2}, \quad G^{C\bar{C}} = \frac{1}{1 - \epsilon^2}. \end{aligned} \quad (54)$$

The G_{ij} elements which are needed for the mass matrix elements are given by

$$\begin{aligned} G_{AA} &= 1, \quad G_{AC} = \epsilon, \\ G_{CC} &= M_P^2 \left[\frac{W_{CC}}{W} - \left(\frac{W_C}{W} \right)^2 \right]. \end{aligned} \quad (55)$$

Since $G_{i\bar{j}}$ are constants, $G_{ij\bar{k}} = 0$. For this reason, the Christoffel symbol $\Gamma_{jk}^i \equiv G^{i\bar{l}} G_{j\bar{k}\bar{l}} = 0$, and hence the Kähler geometry is flat.

The vacuum conditions (i) and (ii) are given by

$$\begin{aligned} G^{i\bar{j}} G_i G_{\bar{j}} &= 3M_P^2, \\ G^{j\bar{k}} G_{\bar{k}} \nabla_i G_j + G_i &= 0. \end{aligned} \quad (56)$$

Since we defined the axion superfield A as the supersymmetrization of Goldstone boson a , the VEV $\langle A \rangle$ is zero

¹ The M term contributes to the saxion derivative in the Lagrangian as $M \partial^2 s$, which has no effect to the axino mass.

as discussed in Sec. II. Now, we calculate for two cases of $\langle C \rangle = 0$ and $\langle C \rangle \neq 0$.

(i) $\langle C \rangle = 0$:

Three vacuum conditions of Eqs. (47) and (48) give the following,

$$\begin{aligned} 2\frac{\alpha}{f_a} &= 0, \\ \frac{1}{1-\epsilon^2} \left| \frac{W_C}{W} \right|^2 &= \frac{3}{M_P^2} \\ \frac{1}{1-\epsilon^2} \left(\frac{W_{CC}}{W} M_P^2 - \epsilon^2 \right) &= 2 \frac{W_C/W}{W_C^*/W^*}. \end{aligned} \quad (57)$$

The first of these requires $\alpha = 0$, and W does not depend on A . Thus, in calculating $G_A = 0$ the $\ln|W|^2$ part of G does not contribute and we cannot constrain the superpotential from the $G_A = 0$ condition.

The axino mass can be obtained from the mass matrix in the (A, C) basis as,

$$\begin{aligned} m &= m_{3/2} \begin{pmatrix} 1 & \epsilon \\ \epsilon & M_P^2 \left[\frac{W_{CC}}{W} - \frac{2}{3} \left(\frac{W_C}{W} \right)^2 \right] \end{pmatrix} \\ &= m_{3/2} \begin{pmatrix} 1 & \epsilon \\ \epsilon & \epsilon^2 \end{pmatrix} \end{aligned} \quad (58)$$

The vacuum conditions determine the (22) element to be ϵ^2 .

To make fermion mass term canonical, we have to re-define fermion as $\psi \rightarrow V\psi$ where

$$V = US = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{1+\epsilon}} & 0 \\ 0 & \frac{1}{\sqrt{1-\epsilon}} \end{pmatrix} \quad (59)$$

and the mass matrix $m = V^T (\nabla_i G_j + (1/3M_P^2) G_i G_j) V$ is given by

$$\frac{m}{m_{3/2}} = \frac{1}{2} \begin{pmatrix} 1+\epsilon & -\sqrt{1-\epsilon^2} \\ -\sqrt{1-\epsilon^2} & 1-\epsilon \end{pmatrix}. \quad (60)$$

Note that $|\epsilon| < 1$ is required to make fermion kinetic term positive definite. The eigenvalues of Eq. (60) are 0 and $m_{3/2}$. This confirms that the vanishing goldstino mass comes out right. The coefficient of $m_{3/2}$ in the axino mass, $\xi_{\text{goldstino}}$, is 1. The axino mass independence of ϵ shows that goldstino is defined in the direction of G_C only.

(ii) $\langle C \rangle \neq 0$:

Here, the vacuum conditions read

$$\begin{aligned} \epsilon(C + C^*) + \frac{\alpha}{f_a} M_P^2 &= 0, \\ \frac{1}{1-\epsilon^2} \left| C + M_P^2 \frac{W_C^*}{W^*} \right|^2 &= 3M_P^2, \\ \frac{1}{1-\epsilon^2} \left[\frac{W_{CC}}{W} M_P^2 - \left(\frac{W_C}{W} \right)^2 - \epsilon^2 \right] &= \frac{C^* + (W_C/W)}{C + (W_C^*/W^*)}. \end{aligned} \quad (61)$$

The left-hand side of the first equation is G_A , and hence SUSY is unbroken in the A direction. But, for C , we have

$$(C + C^*) = -\frac{\alpha}{\epsilon} \frac{M_P^2}{f_a} \simeq -\frac{\alpha}{\epsilon} 10^7 M_P, \quad (62)$$

for $f_a \simeq 10^{-7} M_P$. Hence, for the VEV of C staying at the Planck scale, α needs to be small at $\mathcal{O}(10^{-7})$. Now, the fermion mass matrix is given by

$$\begin{aligned} \frac{m}{m_{3/2}} &= \begin{pmatrix} 1, & \epsilon \\ \epsilon, & M_P^2 \left[\frac{W_{CC}}{W} - \left(\frac{W_C}{W} \right)^2 \right] \\ & + \frac{1}{3} \left(\frac{C^*}{M_P} + M_P \frac{W_C}{W} \right)^2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & \epsilon \\ \epsilon & \epsilon^2 \end{pmatrix} \end{aligned} \quad (63)$$

where the vacuum conditions are used to simplify the (22) element.

We should redefine fermions such that the kinetic terms are canonical, using Eq. (59). The eigenvalues are again 0 and $m_{3/2}$, confirming the correct goldstino mass eigenvalue. The coefficient of $m_{3/2}$ in the axino mass, $\xi_{\text{goldstino}}$, is 1 again.

2. Case for $G_A \neq 0$

In *Case 1*, we considered $K = \frac{1}{2}(A + A^*)^2 + CC^* + \epsilon(A + A^*)(C + C^*)$, which gave $G^A \neq 0$ even though $G_A = 0$ from the vacuum condition. To investigate the case of $G_A \neq 0$ from the beginning with no mixing with SUSY breaking coaxino C , Fig. 4, let us consider

$$\begin{aligned} K &= f_a^2 \left[e^{c_1(A+A^*)/f_a} + e^{-c_2(A+A^*)/f_a} \right] + CC^* \\ W &= e^{\alpha A/f_a} W(C). \end{aligned} \quad (64)$$

Then, with $A = 0$, we obtain

$$\begin{aligned} K_A &= f_a \left[c_1 e^{c_1(A+A^*)/f_a} - c_2 e^{-c_2(A+A^*)/f_a} \right] \\ &= f_a (c_1 - c_2), \\ K_C &= C^* \end{aligned} \quad (65)$$

and

$$\begin{aligned} K_{AA} &= \left[c_1^2 e^{c_1(A+A^*)/f_a} + c_2^2 e^{-c_2(A+A^*)/f_a} \right] \\ &= (c_1^2 + c_2^2), \\ K_{AC} &= K_{CC} = 0. \end{aligned} \quad (66)$$

The Kähler metric is given by

$$\begin{aligned} K_{A\bar{A}} &= \left[c_1^2 e^{c_1(A+A^*)/f_a} + c_2^2 e^{-c_2(A+A^*)/f_a} \right] \\ &= (c_1^2 + c_2^2) \\ K_{A\bar{C}} &= K_{C\bar{A}} = 0, \\ K_{C\bar{C}} &= 1, \end{aligned} \quad (67)$$

and its inverse is given by

$$K^{A\bar{A}} = \frac{1}{K_{A\bar{A}}}, \quad K^{C\bar{C}} = 1, \quad K^{A\bar{C}} = K^{C\bar{A}} = 0. \quad (68)$$

Since

$$\begin{aligned} K_{AA\bar{A}} &= \frac{1}{f_a} \left[c_1^3 e^{c_1(A+A^*)/f_a} - c_2^3 e^{-c_2(A+A^*)/f_a} \right] \\ &= \frac{1}{f_a} (c_1^3 - c_2^3), \end{aligned} \quad (69)$$

and other $K_{ij\bar{k}}$'s vanish, the only nonzero Christoffel symbol is

$$\Gamma_{AA}^A = \frac{1}{f_a} \left(\frac{c_1^3 - c_2^3}{c_1^2 + c_2^2} \right). \quad (70)$$

Now, consider the SUSY breaking. The fermion mass matrix. G_i , the barometer of SUSY breaking, is

$$\begin{aligned} G_A &= f_a(c_1 - c_2) + \frac{\alpha}{f_a} M_P^2 \\ G_C &= C^* + \frac{W_C}{W} M_P^2. \end{aligned} \quad (71)$$

And, G_{ij} in the mass matrix is given by

$$\begin{aligned} G_{AA} &= (c_1^2 + c_2^2) \\ G_{AC} &= 0 \\ G_{CC} &= M_P^2 \left[\frac{W_{CC}}{W} - \left(\frac{W_C}{W} \right)^2 \right]. \end{aligned} \quad (72)$$

Note that $G_{AC} = 0$. It is because of the form of the superpotential. If the superpotential is merely given by a sequestered form, $W = W^{(a)}(A) + W^{(c)}(C)$, the contribution of the superpotential to G_{AC} is

$$\frac{W_{AC}}{W} - \frac{W_A W_C}{W^2} = -\frac{W_A^{(a)} W_C^{(c)}}{W^2} \quad (73)$$

so that it does not vanish in general. However, the shift symmetry of A in G restricts the A dependence of the superpotential to the form $W(C) e^{\alpha A/f_a}$ so that $(W_{AC}/W) - (W_A W_C/W^2)$ vanishes. This seems not giving $m_{\tilde{a}} = 2m_{3/2}$ of Ref. [32]. The reason is the following.

The vacuum condition $G^{i\bar{j}} G_i G_{\bar{j}} = 3M_P^2$ reads

$$\begin{aligned} \frac{1}{c_1^2 + c_2^2} \left(f_a(c_1 - c_2) + \frac{\alpha}{f_a} M_P^2 \right)^2 \\ + \left| C^* + M_P^2 \frac{W_C}{W} \right|^2 = 3M_P^2 \end{aligned} \quad (74)$$

and the conditions $G^j \nabla_i G_j + G_i = 0$ read

$$\begin{aligned} \frac{(c_1^3 - c_2^3)}{(c_1^2 + c_2^2)^2} \left[(c_1 - c_2) + \frac{\alpha}{f_a} M_P^2 \right] &= 2, \\ M_P^2 \left[\frac{W_{CC}}{W} - \left(\frac{W_C}{W} \right)^2 \right] &= -\frac{C^* + M_P^2 W_C/W}{C + M_P^2 W_C^*/W^*}. \end{aligned} \quad (75)$$

In fact, this is where the vanishing contribution of the superpotential to G_{AC} comes in. Suppose $A = C = 0$ and K_i are negligible as assumed in [32]. Then, $G_i \simeq M_P^2 W_i/W$. Let $\nabla_i G_j = K_{ij} - \Gamma_{ij}^k G_k + M_P^2 [(W_{ij}/W) - (W_i W_j/W^2)] \equiv X_{ij} - M_P^2 (W_i W_j/W^2)$. For $i = A$ where we are interested in, the second vacuum condition reads

$$\begin{aligned} 0 &= G^A \nabla_A G_A + G^C \nabla_A G_C + G_A \\ &= G^A [X_{AA} - M_P^2 \frac{W_A^2}{W^2}] + G^C [X_{AC} - M_P^2 \frac{W_A W_C}{W^2}] + G_A \\ &\simeq G^A X_{AA} + G^C X_{AC} - G_A [G^A G_A + G^C G_C] / M_P^2 + G_A \end{aligned} \quad (76)$$

where $G_i \simeq M_P^2 W_i/W$ is used. Since $[G^A G_A + G^C G_C] = 3M_P^2$ from the first vacuum condition, we have

$$X_{AA} = 2 \frac{G_A}{G^A} - \frac{G^C}{G^A} X_{AC}. \quad (77)$$

The factor 2 in front of (G_A/G^A) is the source of $2m_{3/2}$ of [32]. For this factor, $W_A W_C/W^2$ in $\nabla_A G_C$ plays a crucial role but is vanishing with our potential, and we cannot use the first vacuum condition for the factor 2. So, the PQ invariance in the superpotential does not require $m_{\tilde{a}} \sim 2m_{3/2}$. In fact, Eq. (76) shows how factor 2 of Ref. [32] comes about. The factor 2 in the RHS of (77) is its origin, $m = 2m_{3/2}$. Note that it is based on the formalism of Eq. (73), the result of a sequestered superpotential. This does not hold in our case, as explained below Eq. (73). In our case, the superpotential of the form $e^{\alpha A/f_a} W(C)$ leads to different relations on the mass term and other parameters from those of Ref. [32], where the Kähler potential does not play a crucial role.

Even though our examples with one coaxino show $m_{\tilde{a}} \simeq m_{3/2}$, it does not imply that the axino mass should be the gravitino mass, as commented later below Eq. (82). Moreover, if we put mixing between A and C in the above Kähler potential, $(1/M_P^2)(A + A^*)CC^*$, the axino mass is not the exact gravitino mass. It may be larger or smaller than the gravitino mass, but no huge enhancement beyond of order 1: $m_{\tilde{a}} = \mathcal{O}(1) \times m_{3/2}$.

Furthermore, vacuum conditions imply that

$$f_a \leq \frac{\sqrt{3}}{2} \frac{|c_1^3 - c_2^3|}{(c_1^2 + c_2^2)^{3/2}} M_P \quad (78)$$

which would be a criterion for reasonable PQ scale.

From the vacuum conditions, the fermion mass matrix is given by

$$\begin{aligned} \frac{m_{ij}}{m_{3/2}} &= \nabla_i G_j + \frac{1}{3} G_i G_j \\ &= \begin{pmatrix} -(c_1^2 + c_2^2) & 0 \\ 0 & -e^{2i\omega} \end{pmatrix} + \frac{1}{3} \begin{pmatrix} G_A^2 & G_A G_C \\ G_A G_C & G_C^2 \end{pmatrix} \end{aligned} \quad (79)$$

where $G_C \equiv \mathcal{C}e^{i\omega}$. In the G^i direction, we confirm $m_{ij}G^j = 0$, and we have the massless fermion in the G^i direction, as expected.

The canonical kinetic terms can be obtained by rescaling,

$$V = \begin{pmatrix} \frac{1}{\sqrt{c_1^2 + c_2^2}} & 0 \\ 0 & 1 \end{pmatrix}. \quad (80)$$

Then, the physical mass matrix is given by

$$\frac{m_{ab}}{m_{3/2}} = \begin{pmatrix} -1 & 0 \\ 0 & -e^{2i\omega} \end{pmatrix} + \frac{1}{3M_P^2} \begin{pmatrix} G_A G^A & G^A G_C \\ G^A G_C & G_C G^C \end{pmatrix} \quad (81)$$

since $G^A = (1/c_1^2 + c_2^2)G_A$ and $G^C = G_C$. Then, the axino mass Eq. (73) should be modified to

$$\begin{aligned} \frac{m_{\tilde{a}}}{m_{3/2}} &= -[1 + e^{2i\omega}] + \frac{1}{3M_P^2}(G_A G^A + G_C G^C) \\ &= -e^{2i\omega} \end{aligned} \quad (82)$$

since $G_A G^A + G_C G^C = 3M_P^2$. By the phase redefinition, we obtain $m_{\tilde{a}} = m_{3/2}$.

However, when $c_1 = c_2 \equiv c$, the situation changes drastically. The vacuum condition requires $G_A = (\alpha/f_a) = 0$ so that axino does not break SUSY and is completely decoupled from the SUSY breaking field $Z = C$. So, the axino is still massless. It may be a low energy effective description of $W = Z_1(S_1 S_2 - f_1^2) + Z_2(S_1 S_2 - f_2^2)$ discussed in [16]. Even though SUSY is broken in this case, the axion superfield does not appear in the superpotential. Since the axion superfield and the SUSY breaking superfield are decoupled in both Kähler potential (since canonical Kähler is assumed) and superpotential, axino is massless since axion is massless.

Physics of the PQ symmetry breaking can introduce another scale of axino mass. Suppose we regard φ as a dynamical superfield such that Kähler potential is given by, for example,

$$K = \frac{1}{2}\varphi\varphi^* \left[e^{c(A+A^*)/f_a} + e^{-c(A+A^*)/f_a} \right]. \quad (83)$$

One may assume that superpotential depends on C and φ separately, $W = e^{\alpha A/f_a}(W_1(\varphi) + W_2(C))$. In this case, we have to consider 3×3 fermion mass matrix in the basis (φ, C, A) and find that three eigenvalues are zero, $\mathcal{O}(f_a)$, and $\mathcal{O}(m_{3/2}) + \mathcal{O}(m_{3/2}^2/f_a)$, respectively. The last case corresponds to the axino. When the leading order term $\mathcal{O}(m_{3/2})$ is suppressed from some accidental symmetry, the next leading order $\mathcal{O}(m_{3/2}^2/f_a)$ is the axino mass scale.

B. The KSVZ model

In the KSVZ approach, one introduces the heavy quark fields Q_L and Q_R in the superpotential as [25],

$$W_{\text{KSVZ}} = m^3 \Theta e^{A/f_a} + f_Q Q_L \overline{Q}_R \varphi e^{A/f_a}. \quad (84)$$

Model	S_1	S_2	Q_L	\overline{Q}_R	H_d	H_u	q_L	D_R^c	U_R^c
KSVZ	1	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	0
DFSZ	1	-1	0	0	-1	-1	ℓ	$1-\ell$	$1-\ell$

TABLE I: The PQ charge assignment Q . Q_L and \overline{Q}_R denote new heavy quark multiplets.

The PQ symmetry is given near the ϵ point of Fig. 1, with $\Gamma(\overline{Q}_L) = -1/2, \Gamma(Q_R) = -1/2$, and $\Gamma(X) = 1$. Near ϵ , there is no φ type field. But near N_{DW} , $\Gamma(\overline{Q}_L)$ and Q_R are not of the φ type, only X is a φ type field, and Q obtains the heavy quark mass $m_Q = f_Q \langle \varphi(X) \rangle$.

It can be rephrased as follows. The heavy quark interaction with A after integrating out heavy scalars by $\varphi = f_a$, we have the interaction $m_Q Q_L \overline{Q}_R e^{A/f_a}$. Technically, we lose the PQ quantum number information of heavy quarks since they do not have a φ type component but only the phase dependence by the original PQ charges. These phases can be rotated away by redefining the phases of Q_L and Q_R . This heavy quark interaction with A generates the two loop mass presented in Fig. 5 at the order of 10 GeV.

C. The DFSZ model

In the DFSZ framework, the $SU(2)_L \times U(1)_Y$ Higgs doublets carry PQ charges and thus the light quarks are also charged under $U(1)_{\text{PQ}}$ [26]. The charge assignment is shown in Table I. So, the superpotential is written as

$$W_{\text{DFSZ}} = W_{\text{PQ}} + \frac{f_s}{M_P} S_1^2 H_d H_u, \quad (85)$$

where $H_d H_u \equiv \epsilon_{\alpha\beta} H_d^\alpha H_u^\beta$. Integrating out S_1 , we have

$$\begin{aligned} W_{\text{DFSZ}} &= \mu e^{2A/f_a} \varphi(H_d) \varphi(H_u) e^{-2A/f_a} \\ &\quad + f_u q_L e^{\ell\theta} u_R^c e^{(1-\ell)\theta} \varphi(H_u) e^{-A/f_a} \\ &\quad + f_d q_L e^{\ell\theta} d_R^c e^{(1-\ell)\theta} \varphi(H_d) e^{-A/f_a}. \end{aligned} \quad (86)$$

For the quarks, they do not contain the φ type fields since they do not contribute to V_a^2 of Eq. (19) and their phase is just a phase parameter θ . This θ can be removed by redefining the phases of quarks, and we obtain

$$W_{\text{DFSZ}} = \mu \frac{v_u v_d}{2} + (m_t t_L t_R^c + m_b b_L b_R^c + \dots) e^{A/f_a}, \quad (87)$$

with $N_{\text{DW}} = 6$. Equation (87) breaks the PQ symmetry and the axion coupling to stop is

$$\begin{aligned} &M_P^4 \ln \left(\frac{1}{2M_P^6} \mu v_u v_d m_t t_L t_R^c e^{A/f_a} + \text{h.c.} \right) \\ &\rightarrow \frac{1}{M_P^2} \mu v_u v_d m_t |\langle t_L t_R^c \rangle| \cos \frac{a}{f_a} \end{aligned} \quad (88)$$

whose coefficient is much smaller than the QCD instanton contribution Λ_{QCD}^4 . So we can neglect the constant term for the axino mass even though it breaks the PQ symmetry. Also, the constant term does not contribute to the axino mass. Including the SUSY breaking auxiliary field Θ , we consider

$$W_{\text{DFSZ}} = \mu \frac{v_u v_d}{2} + \left(m^3 \Theta + m_t t_L t_R^c + m_b b_L b_R^c + \dots \right) e^{A/f_a}. \quad (89)$$

Through the quark mass terms, we obtain the axino mass as shown in Fig. 5 with Q replaced by the SM quarks. The SM quark mass is at most $m_t/m_Q \sim 10^{-9}$ smaller than that of the heavy quark and we obtain the axino mass in the range 10 eV. Only the SUSY breaking soft mass contribution $m^3 \Theta$ can contribute to the axino mass.

IV. CONCLUSION

After properly defining the goldstino and axion multiplets, we presented the calculational scheme of the axino

mass in the most general framework. For only two light superfields of goldstino and axino, we obtain $\xi_{\text{goldstino}}$ of Eq. (43). For $G_A = 0$ where $G = K + \ln|W|^2$, we obtained $m_{\tilde{a}} = m_{3/2}$ with the axino-gravitino mixing parameter ϵ in the Kähler potential. For $G_A \neq 0$, we showed that the axino mass depends on the details of the Kähler potential. But there is another parameter proportional to the gaugino masses, and we can take a wide range of the axino mass for cosmological applications [1–15]. If the gravity mediation is the dominant one, the axino mass is probably greater than the gravitino mass, but its decay to gravitino is negligible. Still, it softens the cosmological gravitino problem [33] somewhat as discussed in Ref. [3].

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